On reliability-based design of rock tunnel support

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Abstract

Tunneling involves large uncertainties. Since 2009, design of rock tunnels in European countries should be performed in accordance with the Eurocodes. The main principle in the Eurocodes is that it must be shown in all design situations that no relevant limit state is exceeded. This can be achieved with a number of different methods, where the most common one is design by calculation. To account for uncertainties in design, the Eurocode states that design by calculation should primarily be performed using limit state design methods, i.e. the partial factor method or reliability-based methods. The basic principle of the former is that it shall be assured that a structure's resisting capacity is larger than the load acting on the structure, with high enough probability. Even if this might seem straightforward, the practical application of limit state design to rock tunnel support has only been studied to a limited extent.

The aim of this licentiate thesis is to provide a review of the practical applicability of using reliability-based methods and the partial factor method in design of rock tunnel support. The review and the following discussion are based on findings from the cases studied in the appended papers. The discussion focuses on the challenges of applying fixed partial factors, as suggested by Eurocode, in design of rock tunnel support and some of the practical difficulties the engineer is faced with when applying reliability-based methods to design rock tunnel support.

The main conclusions are that the partial factor method (as defined in Eurocode) is not suitable to use in design of rock tunnel support, but that reliability-based methods have the potential to account for uncertainties present in design, especially when used within the framework of the observational method. However, gathering of data for statistical quantification of input variables along with clarification of the necessary reliability levels and definition of "failure" are needed.

Keywords
Rock engineering, reliability-based design, Eurocode 7, observational method, tunnel engineering

Sammanfattning

Tunnelbyggande medför stora osäkerheter. Sedan 2009 kan dimensionering av bergtunnlar utföras i enlighet med Eurokoderna. Grundprincipen i Eurokoderna är att i samtliga dimensioneringsfall skall visas att inget relevant gränstillstånd överskrids. Detta kan uppfyllas genom användningen av ett antal olika metoder där den vanligaste är dimensionering genom beräkning. För att ta hänsyn till osäkerheter vid dimensionering föreskriver Eurokoderna att dimensionering genom beräkning skall utföras med hjälp av gränstillståndsanalys, d.v.s. analys med tillförlitlighetsbaserade metoder eller partialkoefficientmetoden. Grundprincipen för gränstillståndsanalys är att det skall säkerställas att en konstruktions hållfasthet, med tillräckligt hög sannolikhet, är större än lasten som verkar mot konstruktionen. Även om detta kan förefalla enkelt så har den praktiska användningen av gränstillståndsanalys endast studerats i begränsad utsträckning.

Målet med den här licentiatuppsatsen är att bistå med en analys av den praktiska användningen av tillförlitlighetsbaserad analys och partialkoefficientmetoden för dimensionering av bergtunnlars förstärkning. Analysen och den efterföljande diskussionen baseras på det som identifierats i de studerade fallen i de bifogade artiklarna. Diskussionen fokuserar i huvudsak på utmaningarna med att använda de av Eurokoderna föreslagna fasta partialkoefficienterna vid dimensionering av bergtunnelförstärkning samt de praktiska svårigheterna som en ingenjör utsätts för vid användningen av tillförlitlighetsbaserade metoder vid dimensionering av bergtunnelförstärkning.

Slutsatserna som dras är att partialkoefficientmetoden, som den definieras i Eurokoderna, inte är lämplig att använda vid dimensionering av bergtunnelförstärkning men att tillförlitlighetsbaserade metoder har potentialen att ta hänsyn till de osäkerheter som finns vid dimensionering. Detta gäller speciellt om de används inom ramen av observationsmetoden. Dock måste statistiska data för kvantifiering av indatavariabler samlas in och den nödvändiga tillförlitlighetsnivån samt definitionen av "brott" förtydligas.

Nyckelord

Bergmekanik, sannolikhetsbaserad dimensionering, Eurokod 7, observationsmetoden, tunnelbyggnation.

Preface

The research presented in this licentiate thesis was performed between the end of 2015 and the beginning of 2017 at the Division of Soil and Rock Mechanics, Department of Civil and Architectural Engineering, at KTH Royal Institute of Technology in Stockholm, Sweden.

The work was supervised by Professor Stefan Larsson, Dr. Fredrik Johansson, and Dr. Johan Spross. I owe them all much gratitude for their friendship, support, encouragement, and valuable contributions to my work. I would also like to acknowledge my colleagues and friends at the Division of Soil and Rock Mechanics, in particular the co-authors to my research papers: Anders Prästings and Professor Emeritus Håkan Stille, for many rewarding discussions. The input from my reference group is also gratefully acknowledged.

In addition, I would like to acknowledge my colleagues at Skanska's geotechnical division for their friendship, support, and engagement in many interesting discussions.

Further, I would also like to thank my family, especially my parents, Rolf and Christina, and my partner in life, Sandra, for their constant and tireless support.

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Stockholm, April 2017

William Bjureland

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Agne Sandberg Foundation and KTH-Vs foundation are also gratefully acknowledged for supporting scholarship and traveling grant.

List of appended papers

Paper A

Bjureland, W., Spross, J., Johansson, F., Prästings, A. & Larsson, S. 2017. Challenges in applying fixed partial factors to rock engineering design. Accepted by the Geo-Institute *(Geo-Risk) 2017, Denver, Colorado, 4-7 June 2017.*

The author of the thesis performed the calculations and wrote the paper. Spross, Johansson, Prästings and Larsson assisted with comments on the writing.

Paper B

Bjureland, W., Spross, J., Johansson, F. & Stille, H. 2015. Some aspects of reliability-based design for tunnels using observational method (EC7). In: *Kluckner S, ed. EUROCK 2015. 1st ed. Salzburg; 2015:23-29.*

The author, Spross, and Johansson extended a methodology initially proposed by Stille, H. and Holmberg, M. The author performed the calculations and wrote the paper. Spross, Johansson, and Stille assisted with comments on the writing.

Paper C

Bjureland, W., Spross, J., Johansson, F., Prästings, A. & Larsson, S. 2016. Reliability aspects of rock tunnel design with the observational method. Submitted to *International Journal of Rock Mechanics and Mining Sciences.*

The author, Spross, and Johansson extended the methodology proposed in Paper B. The author performed the calculations and wrote the paper. Spross, Johansson, Prästings, and Larsson assisted with comments on the writing.

Other publications

Within the framework of this research project, the author of the thesis also contributed to the following publications. However, they are not included in this thesis.

Johansson, F., Bjureland, W. & Spross, J. 2016. Application of reliability-based design methods to underground excavation in rock. BeFo report 155. *Stockholm: BeFo report 155 (In press).*

Prästings, A., Spross, J., Müller, R., Larsson, S., Bjureland, W. & Johansson, F. 2016. Implementing the extended multivariate approach in design with partial factors for a retaining wall in clay. Accepted in *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering Special Collection SCO23A.*

Sjölander, A. Bjureland, W & Ansell, A. 2017. On failure probability in thin irregular shotcrete shells. Accepted by the International Tunneling and Underground Space Association (*World Tunnel Congress) 2017, Bergen, Norway, 9-15 June 2017.*

Contents

1 Introduction

1.1 Background

In both cities and rural areas, tunnels and caverns are excavated for a number of purposes. In cities, tunnels and caverns are mainly built for infrastructure purposes, such as metro lines, roads, railways, and sewage systems. In more rural areas, underground facilities are excavated for other applications as well, such as hydropower plants, mines, and nuclear waste deposits. Regardless of its location and intended application, underground excavations in rock have the common feature that they involve large uncertainties that must be efficiently accounted for to ensure an environmentally and economically optimized structure that fulfills society's requirements of structural safety.

Design of underground structures in rock can be performed with a number of rock engineering design tools , e.g. classification systems, the New Austrian Tunneling Method (NATM), numerical or analytical calculations, the observational method, and engineering judgement (Palmstrom & Stille 2007). Depending on the expected ground behavior and its connected uncertainties, e.g. phenomenological, model, prediction, physical, and statistical uncertainties (see section 2.5 for description), different tools and safety assessment methods are suitable to use in the analysis.

Historically, design using calculations in combination with the deterministic safety factor approach for safety assessment has played an important role in design codes for management of uncertainties and verification of structural safety. Since 2009, verification of structural safety in civil engineering shall, according to the European commission, in European countries be performed in accordance with the new European design standards, the Eurocodes (CEN 2002). The Eurocodes consist of ten European design standards applicable to most structures and materials of civil engineering: some examples are basis of design (EN1990), concrete (EN1992), steel (EN1993), and soils and rock (EN1997).[1]

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[1] The last digit in the designation of each standard refers to the number of that particular Eurocode, e.g. EN1997 refers to Eurocode 7.

In Sweden, however, design of underground structures in rock is currently, by responsible authorities, exempt from the requirement of using the Eurocodes, since it is unclear to what extent the Eurocodes are applicable to rock engineering. Instead, individual governmental bodies have the possibility to prescribe, within their respective area of responsibility, how design of underground facilities should be performed and if the Eurocodes are applicable. As an example, the Swedish Traffic Administration provides specified recommendations and guidelines for design of road and railway tunnels, according to which the Eurocodes can be used if they can be shown that they are applicable (Lindfors et al. 2015). In addition, work is currently being undertaken to incorporate rock engineering more extensively in the updated version of Eurocode 7, which is due in 2020. This implies that design of underground facilities in rock, within the EU, in the future likely shall be performed in accordance with the Eurocodes.

The basic rule in the Eurocodes is that for all design situations it must be verified that no relevant limit state is exceeded. In each Eurocode, a number of different accepted design tools, or limit state verification methods, are specified. In EN1990 (CEN 2002) the specified methods are structural analysis and design assisted by testing. In Eurocode 7 (CEN 2004), the specified limit state verification methods are geotechnical design by calculation, design by prescriptive measures, load tests and tests on experimental models, and the observational method (Figure 1.1). Limit state verification for the design of underground excavations in rock, can in many situations be performed using calculations (Palmstrom & Stille 2007). For limit state verification with calculation, Eurocode 7 suggests that analytical, semi-empirical, or numerical calculation models are appropriate (Figure 1.1).

To account for physical and statistical uncertainty, the Eurocodes recommend that calculation models are accompanied by a safety assessment using “the partial factor method” to verify limit states. The partial factor method is originally a reliability-based design method that accounts for uncertainties by increasing the calculated load and decreasing the calculated resistance through application of partial factors on their respective characteristic values. The increased load and decreased resistance are usually referred to as design values and structural safety is assured by verifying that the design value of the load is smaller than the design value of the resistance. Thereby, a margin of safety is created against limit state exceedance that has the potential to

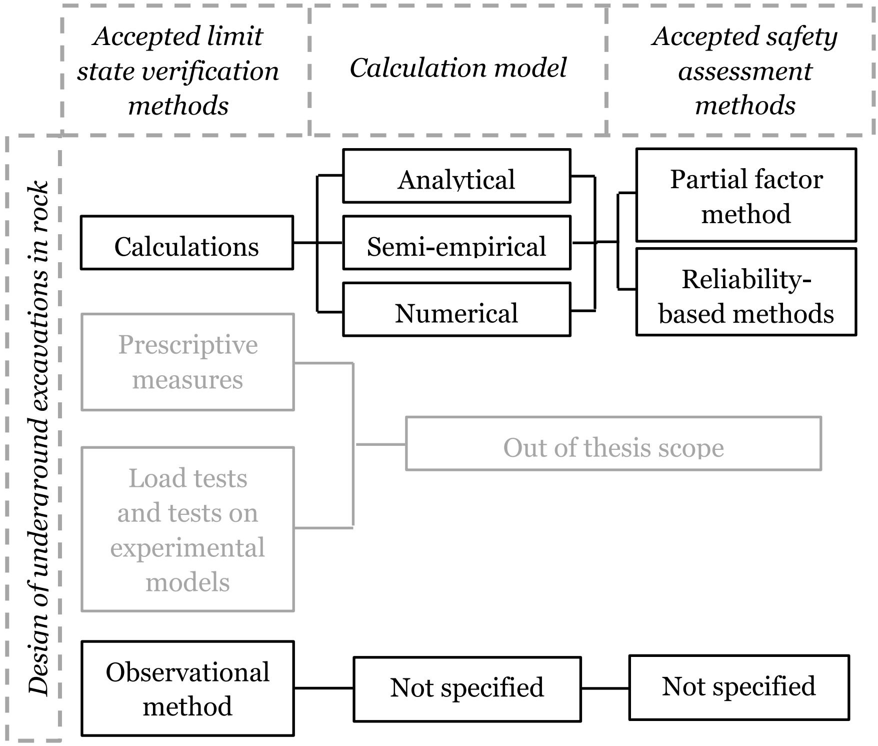

Figure 1.1: Accepted limit state verification tools available to the rock engineer along with suggested calculation models and accepted safety assessment methods.

account for uncertainty of parameters, sensitivity of the analyzed limit state to specific parameters, and the target reliability of the structure. However, in the Eurocodes' version of the partial factor method, fixed partial factors for specific materials are specified.[2] Thus, the aforementioned advantages are possibly lost. As an alternative to the

[2] The partial factor method was originally a reliability-based method applicable to a wide variety of areas. The Eurocodes version of suggesting fixed partial factors differs somewhat from the original method in which partial factors varies with the load–resistance relationship and the magnitude and uncertainty of input parameters. Therefore, in this thesis the partial-factor method, as defined in Eurocode, is not included in the term "reliability-based methods" unless otherwise stated.

partial factor method, the Eurocodes accept the use of reliability-based design methods directly. In reliability-based design methods, uncertainties are accounted for by using the statistical distribution of all relevant input parameters to calculate the probability of limit exceedance, i.e. the probability that the load will exceed the resistance. For every possible limit state, it must be shown that the calculated probability of limit exceedance is sufficiently low. However, similarly to the partial factor method, reliability-based design methods primarily account for physical and statistical uncertainty in input variables. Therefore, limit state verification for underground excavations in rock might not be suitable to perform through calculations solely.

For instance, for ground behaviors that include large epistemic (unknown) uncertainties, such as calculation model and prediction uncertainty, the observational method might be preferable. In the observational method, the main idea is to predict the behavior of a structure, before construction is started, and through monitoring during construction assess the structure's behavior (see Chapter 3). However, as opposed to design by calculations, Eurocode 7 (CEN 2004) gives no recommendations, or limitations, on how the requirements of the observational method stated in Eurocode 7 (CEN 2004) can be fulfilled.

It is clear, however, from the requirements of the observational method that incorporation of calculations, which stringently account for physical and statistical uncertainty in variables, are needed in order to fulfill them. Therefore, to account for and decrease as many uncertainties as possible, present in the design of underground excavations in rock, an attractive approach would be to use reliability-based calculations within the framework of the observational method.

1.2 Project and thesis aim

The overall aim of this project is to develop reliability-based design methods for environmental and economical optimization of rock support in underground excavations.

Taking a reliability-based perspective, this licentiate thesis examines the applicability of design by calculations and design with the observational method. The aim of the study is to identify possibilities and practical difficulties of using the partial factor method and reliability-based methods, exclusively or in combination with the observational method, for design of rock tunnel support. By doing so, optimization of rock tunnel support with respect to the present uncertainties might be possible without compromising on society's requirements of structural safety.

1.3 Outline of thesis

The review performed in this thesis is based on a literature study and the findings from the appended papers.

The performed review in the thesis essentially consists of four chapters covering reliability-based methods in general, the observational method, different aspects of design of rock tunnel support, and some aspects on design of rock tunnel support using reliability-based methods and the observational method. A summary of each of the appended papers is made in the sixth chapter along with a discussion about the implications in the seventh chapter. Lastly, concluding remarks are presented together with suggestions for future research.

1.4 Limitations

Prescriptive measures, load tests, and tests on experimental models, are accepted limit state verification methods according to Eurocode 7 (CEN 2004). However, they are all out of the scope of this thesis.

2 Reliability-based methods

2.1 Factors of safety and limit state design

To account for uncertainties in rock engineering, historically the deterministic safety factor approach has been applied. The safety factor, SF, is usually defined as the ratio between the mean resistance, μ_{R}, of a structure and the mean load, μ_{S}, acting on it:

$$SF = \frac{\mu_{\mathrm{R}}}{\mu_{\mathrm{S}}}. \tag{1}$$

In deterministic design, the idea is that the resistance of a structure must be greater, by a certain factor, or a SF, than the expected load acting on the structure. By doing so, uncertainty in their respective magnitude can be accounted for. The magnitude of the required SF for different limit states is usually determined heuristically, e.g. based on a long experience of similar successful, or unsuccessful, projects. This approach to determine the required SF has led to a situation where the required SF for a certain limit state might not, in design codes and guidelines, be calibrated against society's required levels of safety.

To overcome this issue, the authors of the Eurocodes (CEN 2002) have chosen to apply a different approach and instead use limit state design to account for uncertainty in design of structures. As mentioned in Section 1.1, the preferred limit state design method according to the general Eurocode (CEN 2002) is the partial factor method. The partial factor method's utilization in civil engineering originates from work performed in mid-1900s by structural engineers, such as Freudenthal (1947). At that time, the structural engineers started to question the deterministic design approach's ability to account for the uncertainties present in design. Instead, e.g. Freudenthal (1947) and others began to use reliability-based methods to link structural failure to uncertainty in both load and resistance . This led to the possibility of using reliability-based methods to account for uncertainties by defining a limit state function, G, as the limit between safe and unsafe behavior

$$G(\boldsymbol{X}) = 0, \tag{2}$$

where $\boldsymbol{X}$ is vector that contains all relevant random variables; in its most simple form $G(\boldsymbol{X}) = R - S$, in which R is the resistance and S is the load.

Probability of limit exceedance, p_{f}, i.e. the probability of an unwanted behavior is defined as

$$p_{\mathrm{f}} = P(G(\boldsymbol{X}) \leq 0) = \Phi(-\beta). \tag{3}$$

For a normally distributed $G(\boldsymbol{X})$ the corresponding reliability index, β, is defined as

$$\beta = \frac{\mu_G}{\sigma_G} \tag{4}$$

in which Φ is the cumulative standard normal distribution and μ_G and σ_G are the mean and standard deviation of G, respectively. β is thereby a measure of the distance from the μ_G to the origin, $G(\boldsymbol{X}) = 0$, measured in σ_G (Figure 2.1).

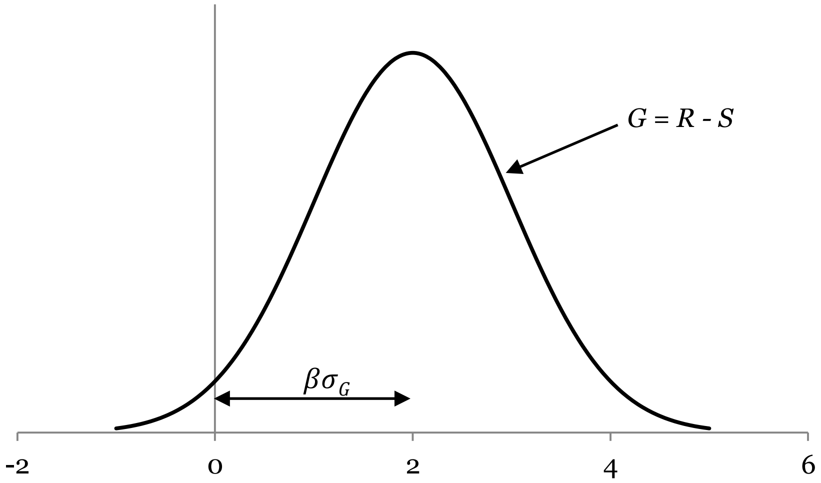

Figure 2.1: Example showing a normal distribution with $\mu_{\mathrm{G}} = 2$, $\sigma_{\mathrm{G}} = 1$, and consequently, $\beta = 2$.

2.2 Frequentist, Bayesian, and nominal views on probability

In structural design, a common interpretation of probability is to judge the frequency of occurrence of an event in an uncertain situation. Such an interpretation is appropriate in many situations. However, there are situations in which it is not. As an example, consider a situation in which a tunnel is planned to be excavated through a fault zone. The client asks the consultant to judge the probability that the fault zone is water bearing and that a large ingress of water into the tunnel is to be expected. In such a situation there is information available but not in terms of frequencies, since it is a one-time event, from which a subjective degree of belief can be asserted. From a purely mathematical or scientific point of view, subjective degrees of belief might be considered irrelevant. However, one often has to make decisions in uncertain situations and a systematic way of using subjective degrees of beliefs is at least a consistent way of making those decisions (Bertsekas & Tsitsiklis 2002).

Making use of subjective degrees of beliefs is the core of the Bayesian interpretation of probability. The Bayesian interpretation is, in that sense, wider than the frequentist interpretation, because it allows for incorporation of both objective data and subjective beliefs in the analysis (Johansson et al. (In press), Vrouwendeler 2002).

In practice, however, a relatively common interpretation of probability of limit exceedance is the nominal one. In the nominal interpretation, it is acknowledged that some approximations and simplifications have been made in the calculated probability of limit state exceedance and that some known uncertainties are left unaccounted for. When these issues are ignored the calculated probability of limit exceedance has no connection to the reliability of the structure, i.e. the calculated probability becomes nominal (Melchers 1999). However, even if the calculated probability becomes nominal it can, if calibrated, be used as a basis for decision making.

As argued for by other authors (e.g. Baecher & Christian 2003, Johansson et al. (In press), Vrouwendeler 2002)) the Bayesian interpretation is the most useful interpretation of probability. Compared to the nominal interpretation the Bayesian interpretation requires that all uncertainties are described and accounted for as accurately as possible, based on the information available to the designer. For this reason, the Bayesian view on probability is used in this thesis and thereby the term probability is used in the wider sense and should be interpreted as a degree of belief.

2.3 Methods for calculating probabilities of unwanted events

2.3.1 General reliability theory

In the general case, Eq. 3 can be solved by evaluating the multidimensional integral over the unsafe region (Melchers 1999)

$$p_{\mathrm{f}} = P[G(\boldsymbol{X}) \leq 0] = \int \dots \int_{\mathrm{G}(\mathbf{X}) \leq 0} f_{\mathbf{X}}(\boldsymbol{x}) d\boldsymbol{x} \tag{5}$$

in which $f_{\mathbf{X}}(\boldsymbol{x})$ is a joint probability density function that contains all random variables. This integral is for most cases very difficult, or even impossible, to solve analytically. Therefore, a number of methods that approximate the integral in Eq. 5 have been developed. These methods are usually divided into three, or four, different levels based on their approach of accounting for uncertainties in input variables. Melchers (1999) uses the following categorization of the different approaches:

- Level I methods account for uncertainty by adding partial factors or load and resistance factors to characteristic values of individual uncertain input variables. Two examples are the partial factor method and the load and resistance factor design.

- Level II methods account for uncertainty through the mean, μ, standard deviation, σ, and correlation coefficients, ρ, of the uncertain random input variables. However, the methods assume normal distributions. Examples of these methods are simplified reliability index, and second-moment methods.

- Level III methods account for uncertainty by considering the joint distribution function of all random parameters. One example of a Level III method is Monte Carlo simulations.

- Level IV methods add the consequences of failure into the analysis, thereby providing a tool for, e.g., cost–benefit analyses.

As the forth level includes consequences, it is sometimes excluded in the categorization of the different methods.

2.3.2 The partial factor method

The partial factor method is a limit state design method that accounts for uncertainties by applying a partial factor to the characteristic values of load and resistance. It is the preferred design method in Eurocode (CEN 2002); however, the Eurocodes version is slightly adjusted from the original method.

In the original method, partial factors have a clear connection to reliability-based design. In the original method, partial factors are statistically derived for both load and resistance, respectively, from the general expressions (Melchers 1999)

$$\gamma_{\mathrm{S,j}} = \frac{x_{\mathrm{d},j}}{x_{\mathrm{k},j}} = \frac{F_{\mathrm{X}_j}^{-1}[\Phi(y_j^*)]}{x_{\mathrm{k},j}} \tag{6}$$

and

$$\gamma_{\mathrm{R},i} = \frac{x_{\mathrm{k},i}}{x_{\mathrm{d},i}} = \frac{x_{\mathrm{k},i}}{F_{\mathrm{X}_i}^{-1}[\Phi(y_i^*)]} \tag{7}$$

in which $x_{\mathrm{k},i}$ and $x_{\mathrm{k},j}$ represents characteristic values; $x_{\mathrm{d},i}$ and $x_{\mathrm{d},j}$ are design values that can be found by transforming the coordinates of Hasofer and Lind's (1974) design point, $\boldsymbol{y}^*$, back from standard normal space, Y. This back transformation is denoted $F_{\mathrm{X}_i}^{-1}[\Phi(y_i^*)]$. Principally, $x_{\mathrm{d},i}$ and $x_{\mathrm{d},j}$ are dependent on the variable's mean, μ, the directional cosines (sensitivity factors) α_i, the target reliability index, β_{T}, and coefficient of variation, $CO\mathrm{V}$. Extended presentations of α_i, β_{T}, and $CO\mathrm{V}$ are given in Sections 2.3.3, 2.4, and 2.5, respectively.

In Eurocodes version of the partial factor method, fixed partial factors are proposed for different materials. The proposed values are based on two approaches: a long experience of building tradition (the most common approach in Eurocode), or on the basis of statistical evaluation of experimental data and field observations (CEN 2002).

2.3.3 Second-moment and transformation methods

Second-moment methods started to gain recognition in the late 1960s, based on the work performed by Cornell (1969). The second-moment methods belong to a group of approximate methods that can be used to calculate p_{f} by approximation of the integral in Eq. 5 through the first two moments in the random variables, i.e. the mean and standard deviation. However, generally, the $G(\boldsymbol{X})$ is not linear and thereby the first two moments of $G(\boldsymbol{X})$ are not available (Melchers 1999). To solve this, the second-moment methods uses Taylor series expansion about some point, x^*, to linearize $G(\boldsymbol{X})$. Approximations that linearize $G(\boldsymbol{X})$ are usually referred to as "first order" methods (Melchers 1999).

An improvement was proposed by Hasofer & Lind (1974). By transforming all variables to their standardized form, standard normal distribution $N(0,1)$, computation of β becomes independent of algebraic reformulation of $G(\boldsymbol{X})$. This is usually referred to as the "first-order reliability method" (FORM). Further improvements have since then been made for situations such as for non-normal distributions and for correlation between variables (e.g. Hochenbichler & Rackwitz 1981).

In principle the methodology of FORM is as follows. First, all random variables and the limit state function are transformed into Y through:

$$Y_i = \frac{\mathrm{X}_i - \mu_{X_i}}{\sigma_{X_i}}, \tag{8}$$

in which Y_i is the transformed variable X_i with $\mu_{Y_i} = 0$ and $\sigma_{Y_i} = 1$. The μ_{X_i} and σ_{X_i} are the mean and standard deviation of the X_i, respectively (Melchers 1999).

In the Y, the $G(\boldsymbol{Y})$ is a linearized hyperplane from which evaluation of the shortest distance to the origin yields β. This evaluation can be made through:

$$\beta = \min_{\mathrm{G(Y)=0}} \sqrt{\sum_{i=1}^{n} y_i^2}, \tag{9}$$

in which y_i represents the coordinates of any point on the limit state surface, $G(\boldsymbol{Y})$ (Melchers 1999). The point that is closest to the origin is often referred to as the "design point" or "checking point", y^*, and it represents the point of greatest probability for the $g(\boldsymbol{Y}) < 0$ domain.

One very useful feature of FORM is that α_i can be derived. The α_i can be found by first calculating the outward normal vector, c_i, to the $g(\boldsymbol{Y}) = 0$

$$c_i = \lambda \frac{\partial \mathrm{g}}{\partial \mathrm{y}_i}, \tag{10}$$

in which λ is an arbitrary constant, and then calculating the length of the outward normal vector, l,

$$l = \sqrt{\sum_i c_i^2}. \tag{11}$$

α_i is defined as

$$\alpha_i = \frac{c_i}{l} \tag{12}$$

indicating how sensitive $G(\boldsymbol{Y})$ is to changes in the respective Y_i.

2.3.4 Monte Carlo simulations

Monte Carlo simulations are a repetitive numerical process for calculating probability (Ang & Tang 2007). The process starts with generating a random number from the probability density function of each of the predefined random variables, $\hat{x}$. For each repetition, $G(\hat{\boldsymbol{x}})$ is evaluated and for every combination of $\hat{\boldsymbol{x}}$ where $G(\hat{\boldsymbol{x}}) \leq 0$, the limit between the safe and unsafe behavior, defined by G, is exceeded; i.e. the result is deemed as "failure". Repeating the process for a large number of repetitions, counting the number of "failures", and comparing them with the total number of repetitions, N, gives an estimate of p_f.

The accuracy of the calculated p_f is dependent on N and the magnitude of the calculated p_f. In principle, the smaller p_f is the larger N must be to gain the same level of accuracy of the calculated p_f. To find the required number of calculations to achieve a particular level of accuracy, the following can be used (Harr 1987). As each simulation is an experiment with a probability of a successful result, p_s, and a probability of an unsuccessful result, p_u, equal to $1 - p_\mathrm{s}$, assuming that the simulations are independent. Thus, the simulations will yield a binomial distribution with an expected value of Np_s and a standard deviation of $\sqrt{Np_\mathrm{s}(1-p_\mathrm{s})}$. Then if x_su (which will be normally distributed) is defined as the number of successes in N simulations and $x_{\tilde{\alpha}/2}$ as the number of successes in N simulations such that the probability of a value larger or smaller, then that value is not greater than $\tilde{\alpha}/2$, the number of simulations required, N_{req}, is

$$N_{req} = \frac{p_\mathrm{s}(1-p_\mathrm{s})h_{\tilde{\alpha}/2}^2}{e^2}, \tag{13}$$

in which $h_{\tilde{\alpha}/2}$ is the normally distributed quantile for a chosen credibility level and e represents the maximum allowable system error given as

$$e = p_{\mathrm{s}} - \left(\frac{x_{\frac{\tilde{\alpha}}{2}}}{N}\right). \tag{14}$$

As can be seen from Eq. 13, $p_{\mathrm{s}}(1 - p_{\mathrm{s}})$ is maximized when p_{s} is ½. Thereby, a conservative approach is to use $p_{\mathrm{s}}(1 - p_{\mathrm{s}}) = 1/4$, which, for a limit state with a single variable, yields that

$$N_{req} = \frac{h_{\tilde{\alpha}/2}^2}{4e^2} \tag{15}$$

and for a limit state with multiple variables, m,

$$N_{req} = \left(\frac{h_{\tilde{\alpha}/2}^2}{4e^2}\right)^m. \tag{16}$$

2.4 Acceptable probability of unwanted events

When using reliability-based methods, it must be shown that the designed structure fulfills the required levels of safety, as demanded by society. In Eurocode (CEN 2002), society's demands on acceptable levels of safety in ultimate limit state are defined as a target reliability index, β_{T}, or $p_{\mathrm{f,T}}$ with a magnitude that depends on the reliability class of the structure. Required β_{T} values can be seen in Table 2.1.

The reliability class of the structure is in turn respectively related to the consequences of limit state exceedance. Similar to reliability classes, Eurocode (CEN 2002) divides this into three different levels. The consequence classes can be seen in Table 2.2.

Table 2.1: Acceptable levels of safety according to Eurocode.

Reliability class	β_{T}	$p_{\mathrm{f,T}}$
RC1	4.20	$1.33 * 10^{-5}$
RC2	4.70	$1.30 * 10^{-6}$
RC3	5.20	$1.00 * 10^{-7}$

Table 2.2: Definition of consequence classes in Eurocode.

Consequence class	Description	Example
CC1	Small risk of death, and small or negligible economical, societal or environmental consequences.	Farm buildings where people don’t normally reside.
CC2	Normal risk of death, considerable economical, societal or environmental consequences.	Residence and office buildings.
CC3	Large risk of death, or very large economical, societal or environmental consequences.	Stadium stands and concert halls.

2.5 Sources of uncertainties

Similar to both deterministic design and the partial factor method, reliability-based methods are a means of accounting for uncertainties in input variables. Uncertainties are commonly divided into two different types, i.e. *aleatory* or *epistemic*. Aleatory uncertainty is due to the inherent variability, or randomness, in input variables and can therefore not be reduced. Epistemic uncertainty on the other hand is uncertainty that is due to a lack of knowledge and can thereby be reduced, simply by gaining more information (Ang & Tang 2007).

Instead of dividing uncertainties into either aleatory or epistemic, a more detailed breakdown can be made based on the sources of uncertainty. Baecher & Christian (2003) did so by dividing uncertainties into three different categories; characterization uncertainty, model uncertainty, and parameter uncertainty. Characterization uncertainty is related to uncertainty in the interpretation results from site investigations. Model uncertainty relates to uncertainty in the applied calculation model. Parameter uncertainty relates to the uncertainty that might be introduced in the operationalization of a measurement, i.e. the transformation from an observed parameter to an inferred property of interest. The total parameter uncertainty, assuming independence, can then be expressed in COV's (Müller et al. 2013, Goodman 1960, Baecher & Ladd 1997):

$$COV_{\mathrm{tot}}^2 = COV_{\mathrm{sp}}^2 + COV_{\mathrm{err}}^2 + COV_{\mu}^2 + COV_{\mathrm{tr}}^2 , \tag{17}$$

in which COV_{sp} refers to uncertainty introduced by the inherent variability of the property, COV_{err} refers to random error introduced by the measurement, COV_{μ} refers to uncertainty in determination of the mean value of the property, and COV_{tr} refers to uncertainty in possible biases that might be introduced in the operationalization of the studied property.

As an alternative to the categorization made by Baecher & Christian (2003), Melchers (1999) provides an extended division of sources for uncertainties and argues that there are seven main such sources; phenomenological uncertainty, decision uncertainty, modelling uncertainty, prediction uncertainty, physical uncertainty, statistical uncertainty, and uncertainty due to human factors.

2.5.1 Phenomenological uncertainty

Phenomenological uncertainty relates to uncertainty in the phenomena relevant for a structure's expected behavior. It is of particular importance for novel and 'state of the art' techniques in which a structure's behavior during construction, service life, and extreme conditions might be difficult to assess.

2.5.2 Decision uncertainty

Decision uncertainty relates to the decision of whether or not a particular phenomenon has occurred. For limit state design, decision uncertainty purely concerns the decision as to whether limit state exceedance has occurred.

2.5.3 Modelling uncertainty

Modelling uncertainty concerns uncertainty in the applied calculation model, i.e. how well the model represents the physical behavior of the physical structure. Model uncertainty is often due to our lack of knowledge on how to describe the physical behavior of a structure through simplified mathematical relationships.

2.5.4 Prediction uncertainty

Connected to modeling uncertainty is prediction uncertainty, which concerns our ability to predict the future behavior of a structure, e.g. the prediction of expected deformations when a structure is being exposed to loads. Prediction uncertainties can usually be reduced as new knowledge, e.g. during construction, becomes available and the predicted behavior can be refined. In tunnel engineering, reduction of prediction uncertainties, by gaining more information, can be achieved through e.g. observations of rock mass quality during excavation and measurements of stresses and deformations.

2.5.5 Physical uncertainty

Physical uncertainty relates to the inherent variability, or randomness, of the basic variables. Reduction of physical uncertainty can be performed by gaining more information of the basic variables through more field and laboratory tests of rock mass parameters or support characteristics. However, physical uncertainty can usually not be eliminated.

2.5.6 Statistical uncertainty

Statistical uncertainty concerns the determination of statistical estimators to suggest an appropriate probability density function. It arises since assigned probability density functions usually don't perfectly mimic the available data and also when a limited number of tests are available as a basis.

2.5.7 Uncertainty due to human factors

Human errors are those which are due to the natural variation in task performance and those which occur in the process of design, documentation, and construction and use of the structure within accepted processes. In addition, uncertainties due to human errors are those which are a direct result of neglecting fundamental structural or service requirements. Uncertainties due to human factors can usually be reduced through human intervention strategies such as education, good work environment, complexity reduction, personnel selection, self-checking, external checking and inspection, and sanctions.

3 Observational method

The observational method, which is an accepted limit state verification method in Eurocode 7 (CEN 2004), is usually credited to originate from the works performed by Terzaghi and Peck in the early and mid-1900s (Peck 1969), even though successful similar approaches had been used before e.g. the final report by the Geotechnical Committee of the Swedish State Railways (1922). The main idea of the methodology is to predict the behavior, of a geotechnical structure, before the start of construction and during construction, monitor and assess the structure's behavior. The method is similar to the, at least in Sweden, well-known approach called "active design" (Stille 1986).

3.1 The observational method as proposed by Peck

One of the key considerations of Peck's and Terzaghi's formulation of the observational method was to account for uncertainties, for safety and optimization reasons, in design of underground structures. In line with these considerations, Peck (1969) defined a number of elements that must be included in the complete application of the method.

a. "Exploration sufficient to establish at least the general nature, pattern and properties of the deposits, but not necessarily in detail.
b. Assessment of the most probable conditions and the most unfavourable conceivable deviations from these conditions. In this assessment geology often plays a major rôle.
c. Establishment of the design based on a working hypothesis of behaviour anticipated under the most probable conditions.
d. Selection of quantities to be observed as construction proceeds and calculation of their anticipated values on the basis of the working hypothesis.
e. Calculations of values of the same quantities under the most unfavourable conditions with the available data concerning the subsurface conditions.
f. Selection in advance of a course of action or modification of design for every foreseeable significant deviation of the observational findings from those predicted on the basis of the working hypothesis.

g. Measurement of quantities to be observed and evaluation of actual conditions.
h. Modification of design to suit actual conditions.”

3.2 The observational method as defined in Eurocode 7

Similar to Peck (1969), Eurocode demands that certain elements must be included in a successful application of the methodology. These elements are in principle comparable to the elements included in Peck’s suggestion; however, defined slightly different:

“(1) When prediction of geotechnical behavior is difficult, it can be appropriate to apply the approach known as ‘the observational method’ in which the design is reviewed during construction.
(2)P The following requirements shall be met before construction is started:

a) acceptable limits of behavior shall be established;
b) the range of possible behavior shall be assessed and it shall be shown that there is an acceptable probability that the actual behavior will be within the acceptable limits;
c) a plan of monitoring shall be devised, which will reveal whether the actual behavior lies within the acceptable limits. The monitoring shall make this clear at a sufficiently early stage, and with sufficiently short intervals to allow contingency actions to be undertaken successfully;
d) the response time of instruments and the procedures for analyzing the results shall be sufficiently rapid in relation to the possible evolution of the system;
e) a plan of contingency actions shall be devised, which may be adopted if the monitoring reveals behavior outside acceptable limits.

(3)P During construction, the monitoring shall be carried out as planned.
(4)P The results of the monitoring shall be assessed at appropriate stages and the planned contingency actions shall be put into operation if the limits of behavior are exceeded.
(5)P Monitoring equipment shall either be replaced or extended if it fails to supply reliable data of appropriate type or in sufficient quantity.”

The principles marked with “P” must not be violated.

3.3 The use of the observational method in today's tunneling

Even though underground construction was one of the key considerations for the formulation of the observational method, the methodology, as defined in Eurocode 7, for design of underground facilities is rarely utilized in practice (Spross 2016). One reason for this, as argued for by Spross (2016), might be that the inflexible requirements, such as showing that the geotechnical behavior with a sufficient probability will be within the acceptable limits, reduces the possible application of the method. In addition, the lack of guidance on how the requirements can be fulfilled hampers the implementation further.

To increase its applicability, Spross (2016), similar to other authors (e.g. Palmstrom & Stille 2007, Maidl et al. 2011, Zetterlund et al. 2011), suggests that reliability-based methods should be incorporated into the framework of the observational method. By doing so, the reliability-based methods can be used (Spross et al. 2014a, Holmberg & Stille 2007, 2009, Stille et al. 2005b, Spross & Johansson 2017) to perform a preliminary design in which a prediction is made about the structures most probable and possible behavior.

3.4 Conditional probability and Bayes' rule

In addition to the preliminary design, through monitoring of the structure's behavior during the course of construction, the expected behavior can be continuously assessed through the use of Bayesian updating (Spross et al. 2014b, Stille et al. 2003, Stille et al. 2005b, Holmberg & Stille 2007, Miranda et al. 2015).

According to Bayes' rule, the probability of an event A_i, occurring given that an event B has occurred, is (Bertsekas & Tsitsiklis 2002)

$$\begin{aligned} \mathrm{P}(A_i|B) &= \frac{\mathrm{P}(A_i)\mathrm{P}(B|A_i)}{\mathrm{P}(B)} \\ &= \frac{\mathrm{P}(A_i)\mathrm{P}(B|A_i)}{\mathrm{P}(A_1)\mathrm{P}(B|A_1) + \cdots + \mathrm{P}(A_n)\mathrm{P}(B|A_n)}, \end{aligned} \tag{18}$$

in which $\mathrm{P}(B|A_i)$ is the probability of event B occurring conditioned on the fact that event A_i has occurred; which in turn can be found through the conditional,

$$P(B|A_i) = \frac{P(A \cap B)}{P(B)}, \tag{19}$$

and total probability theorems

$$\begin{aligned} P(B) &= P(A_1 \cap B) + \cdots + P(A_n \cap B) \\ &= P(A_1)P(B|A_1) + \cdots + P(A_n)P(B|A_n). \end{aligned} \tag{20}$$

An illustration of how Bayes' rule can be utilized in rock tunnel engineering can be seen in Paper B and Paper C. In the papers, Bayes' rule is used to update the probability of limit exceedance after measurements of deformations have been performed.

4 Design of rock support

4.1 Introduction

In design of underground excavations in rock, there are a number of failure modes, or limit states, that the engineer needs to consider. Depending on e.g. the type of rock mass, the stress conditions, the depth and geometry of the tunnel or cavern, different limit states are relevant.

Limit states can be divided into two main types: (I) limit states in which load and resistance can be, through simplifications, viewed as separable and (II) limit states with interaction between the load and the resistance (Johansson et al. (In press)). In the following sections, some typical limit states of type I and type II are presented.

4.2 Limit states with separable load and resistance

The common feature for limit states of type (I) is that, after simplifications, a distinction can be made between the variables affecting the load and the variables affecting the resistance (Bagheri 2011). For example, consider the limit states, or failure modes, presented in the Swedish Traffic Administration's design guidelines (Lindfors et al. 2015). Some common limit states of Type I are e.g. the suspension of a loose core of rock mass using rock bolts and gravity loaded arch, both of which who are governed by the theory of arching (Johansson et al. (In press)).

4.2.1 Gravity-loaded shotcrete arch for tunnels with limited rock cover

The theory of arching in soil has been studied by numerous authors, most of them through experimental utilization of a horizontal trapdoor (Terzaghi 1936, Ladanyi & Hoyaux 1969, Vardoulakis et al. 1981, Evans 1984, Stone 1988, Adachi et al. 1997, Dewoolkar et al. 2007, Chevalier et al. 2009, Costa et al. 2009, Iglesia et al. 2014). The studies show that if the supporting substructure, i.e. the supporting trapdoor, is displaced, the vertical load acting on the trapdoor will be partly transferred to the sides, which causes an increase in the horizontal stresses, $\Delta\sigma_{\mathrm{hr}}$, acting at the bottom of the arch. In principle, the effective vertical load acting on the trapdoor will then consist of the weight of the soil between the arch and

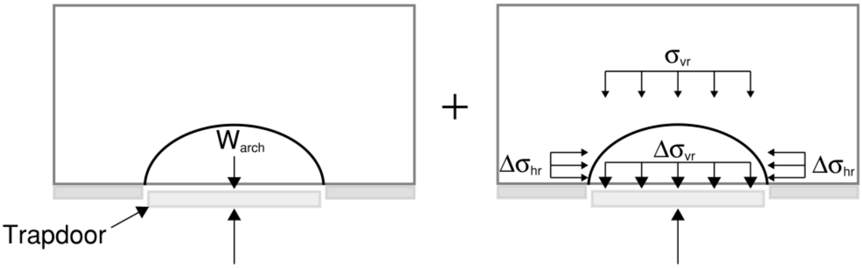

Figure 4.1: The effective vertical load acting on the trapdoor. σ_{vr} is the vertical stress from the surrounding soil acting at the top of the arch. Modified from Iglesia et al. (2014).

the trapdoor, W_{arch}, and increased vertical stresses, $\Delta\sigma_{\mathrm{vr}}$, which are derived from the increased horizontal stresses (Figure 4.1) (Iglesia et al. 2014). The height of the arch depends on the width of the substructure and the horizontal stresses acting at the arch abutments.

The design of a gravity-loaded shotcrete arch has its foundation in the theory of arching. In situations where there is a limited rock cover, it is assumed that a natural arch cannot develop in the rock mass above the tunnel roof, due to the limited rock cover. Thereby the load acting on the supporting shotcrete arch will consist of the load from the above situated soil and rock. The limit state can be analyzed using the following limit state function (Johansson et al. (In press), Lindfors et al. 2015):

$$G = h_{\mathrm{t}}\, f_{\mathrm{cc}}\, t_{\mathrm{c}} - \frac{q_{\mathrm{v}}\, B^2}{8} = 0 \tag{21}$$

in which h_{t} is the height of the tunnel arch, f_{cc} is the compressive strength of the shotcrete, t_{c} is the required shotcrete thickness, B is the width of the tunnel, and q_{v} is the vertical load acting on the shotcrete arch. The required t_{c} can be calculated using either

$$t_{\mathrm{c}} = \frac{2Bq_{\mathrm{v}}}{6.3f_{\mathrm{cc}}}\sqrt{1 + \frac{B^2}{10h_{\mathrm{t}}}} \tag{22}$$

or

$$t_{\mathrm{c}} = \frac{q_{\mathrm{v}}B^2}{8h_{\mathrm{t}}f_{\mathrm{cc}}} \tag{23}$$

depending on if the load acting on the top of the arch is considered to be sinusoidal (Holmgren 1992) or evenly distributed (Stille & Nord 1990), respectively. It should be noted that the above shown equations, are based on moment equilibrium at the top of the tunnel roof. No consideration is made to the fact that the resultant, i.e. the force in the shotcrete, increases with the vertical force towards the abutments of the shotcrete arch. Consequently, the required t_c will be underestimated at the arch abutments.

4.2.2 Suspension of loose core of rock mass

For deeply situated underground facilities in fractured hard rock, problems with arch stability can also occur if a supporting arch cannot develop in the rock mass surrounding the underground excavation. Instability can occur for different reasons: block rotation, sliding along a joint, overstressing of the rock mass, or low horizontal stresses (Stille et al. 2005a, Johansson et al. (In press)). If a stable arch cannot be ensured, the loose core of rock mass must be suspended, e.g. using rock bolts (Figure 4.2). The analysis of the required size and number of rock bolts

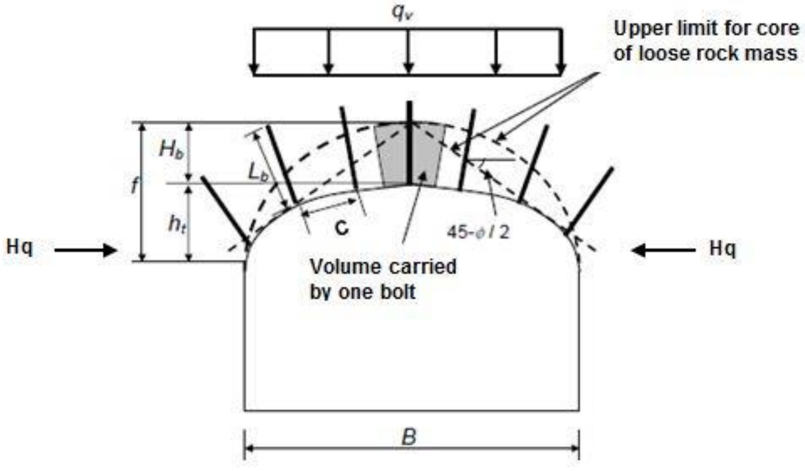

Figure 4.2: Principle figure showing the load case related to the suspension of a loose core of rock mass (Lindfors et al. 2015).

can be performed using the following limit state function (Johansson et al. (In press), Lindfors et al. 2015):

$$G = \frac{\sigma_{\mathrm{y}} A_{\mathrm{s}}}{C^2} - (f_{\max} - H_{\mathrm{b}}) g \rho = 0 \tag{24}$$

in which σ_{y} is the yield strength of the rock bolt steel, A_{s} is the area of the rock bolt, C is the centre to centre distance between the bolts, H_{b} is the height of tunnel roof arch, g is the gravitational acceleration, ρ is the density of the rock mass, and $f_{\max}$ is the maximum peak height of the arch, which can be found by analysing the stress distribution surrounding the tunnel.

If the stress distribution surrounding the tunnel is analyzed, it might be found that the underground excavation, for different reasons, will cause the tangential stresses above the tunnel roof to exceed the compressive strength of the rock mass, which crushes the rock mass in the tunnel roof. In this case, the overstressed rock resembles a uniaxial compressive test in which the failure line in the rock mass could be approximated to slope at an angle equal to $45 - \varphi/2$ (Johansson et al. (In press)), where φ is the friction angle of the rock mass. The peak height of the loose core, f_{o}, caused by the overstressing of the rock mass can under these conditions be calculated through:

$$f_{\mathrm{o}} = \frac{B}{2} \tan\left(45 - \frac{\varphi}{2}\right) \tag{25}$$

On the other hand, when H_{q} is low compared to q_{v}, the result might be that a high compressive arch in the rock mass, with a core of loose unstressed rock below, is identified. The peak height of the unstable arch due to low H_{q}, f_{u}, is given by moment equilibrium as:

$$f_{\mathrm{u}} = B^2 \frac{q}{8H_{\mathrm{q}}} \tag{26}$$

4.2.3 Single block supported by shotcrete

Another common failure mode, when tunnelling in hard crystalline brittle rock, that needs to be accounted for is unstable blocks. The analysis of unstable blocks and the design of support measures to secure them have been studied by numerous authors (e.g. Hoek & Brown 1980, Brady & Brown 2013, Goodman & Shi 1985, Bagheri 2011, Hatzor 1992, Mauldon 1992, Mauldon 1993, Mauldon & Goodman 1996, Mauldon 1990, Mauldon & Goodman 1990, Tonon 1998, Tonon 2007). Depending on the assumptions made, e.g. geometry of the block, failure mode, how to account for stresses in the rock mass, cohesion and friction in the rock joint, the principles of how to perform the analysis differ.

In Sweden, a common support measure is the application of fibre-reinforced shotcrete to the tunnel surface in combination with systematic bolting, i.e. rock bolts are installed in a pre-defined systematic pattern. A potential loose block in between rock bolts is supposed to be secured by the applied shotcrete layer.

According to the Swedish Transport Administration's design guidelines (Lindfors et al. 2015), the analysis of shotcrete's capacity to withstand the load from a loose block differs depending on whether sufficient adhesion, σ_{adk}, in the rock–shotcrete interface develops ($\sigma_{\mathrm{adk}} > 0.5$).

With adhesive contact

If sufficient σ_{adk} in the rock–shotcrete interface develops the design is performed based on the assumption that the load from the block will be carried by the adhesion in the rock–shotcrete interface. Assuming that a block exists, between rock bolts, and neglecting a possible friction in the rock joints, the analysis of the shotcrete's capacity to withstand the load from the block can be performed using the following limit state function (Lindfors et al. 2015, Johansson et al. (In press)):

$$G = \sigma_{\mathrm{adk}} \delta_{\mathrm{m}} O_{\mathrm{m}} - \gamma_{\mathrm{rock}} V_{\mathrm{block}} \geq 0 \tag{27}$$

where δ_{m} is the load-bearing width, O_{m} is the circumferential length of the block, V_{block} is the volume of the block, and γ_{rock} is the unit weight of the rock. Figure 4.3 illustrates the failure mode.

Without adhesive contact

If adhesion in the rock–shotcrete interface can be assumed to be non-existing or if the rock mass is highly fractured, the load from the block must be carried through the moment capacity of the shotcrete. The analysis can be performed using the following limit state function (Lindfors et al. 2015, Johansson et al. (In press)):

$$G = \frac{f_{\mathrm{flr}} t_{\mathrm{c}}^2}{6} - M \geq 0, \tag{28}$$

in which f_{flr} is the bending tensile capacity of the shotcrete, t_{c} is the thickness of the applied shotcrete layer, and M is the bending moment acting on the shotcrete layer. Figure 4.4 shows an illustration of the failure mode.

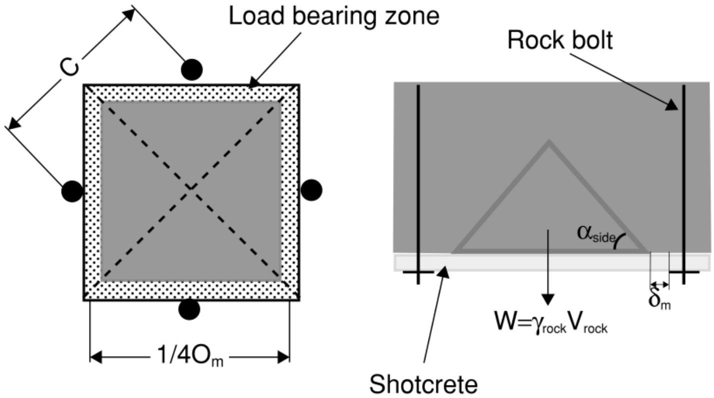

Figure 4.3: The load case related to the analysis of a single block acting on a shotcrete support accounting for adhesion between the shotcrete and the rock mass. *C* is the centre to centre distance between rock bolts, *W* is the total weight of the block, and α_{side} is the angle of the fracture. Modified from Lindfors et al. (2015).

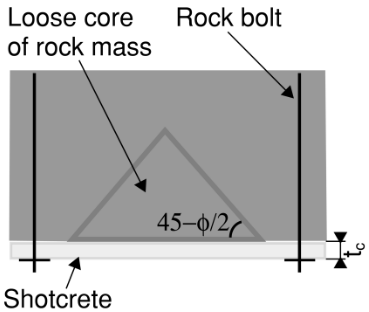

Figure 4.4: The load case related to the analysis of a single block acting on a shotcrete support without accounting for adhesion between the shotcrete and the rock mass. Modified from (Lindfors et al. 2015).

4.3 Limit states with interaction between load and resistance

For limit states of type (II), a clear distinction between the load and the resistance does not exist. As an example, the convergence–confinement method (e.g. Brown et al. 1983), is a typical case in which it might be difficult to derive how different uncertain variables affect the behavior of the analyzed structure. The convergence–confinement method is a graphical solution that describes the development of radial peripheral deformations in a deeply situated circular tunnel with a radius, r, during excavation (Figure 4.5). The deformations develop as a result of the stress changes in the surrounding rock mass. In the following, an elastic–plastic rock mass with a Mohr–Coulomb failure criterion and a non-associated flow rule for the dilatancy after failure is assumed (Stille et al. 1989).

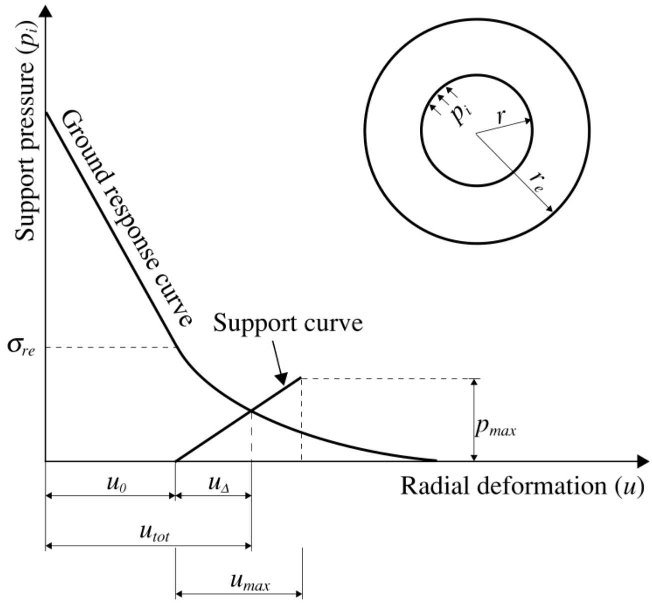

Figure 4.5: Ground and support response curves. u_{max} is the maximum deformation that the shotcrete can withstand, u_0 is the deformation that has developed when the excavation face reaches the considered cross-section, u_Δ is the deformation of the shotcrete, and u_{tot} is the total expected deformation of the tunnel periphery. p_{max} and σ_{re} are defined in the text below.

Illustratively, consider a cross-section along the progression line of a deeply situated circular tunnel. Before excavation is started, a certain initial stress state, p_0, supporting the imaginary periphery of the planned tunnel is present in the rock mass. When excavation has been initiated and the face of the excavation approaches the considered cross-section, the supportive initial stresses starts to decrease. For small changes in the stress state, i.e. at some distance before the excavation reaches the cross section, elastic radial deformations of the tunnel surface, u_{ie}, develop due to the decrease in the supportive radial pressure, p_{i}, acting on the tunnel periphery. The magnitude of the u_{ie} can be calculated as:

$$u_{\mathrm{ie}} = r\frac{1+\nu}{E}(p_0 - p_{\mathrm{i}}), \tag{29}$$

where ν and E are Poisson's ratio and Young's modulus of the rock mass, respectively. When the excavation advances further, p_{i} continues to decrease until eventually the decrement of stresses in the surrounding rock mass reaches a limit, σ_{re}. At this stage, plastic behavior of the rock mass in a zone with radius r_{e} surrounding the tunnel periphery starts to develop (Fig. 4.4). σ_{re} can be calculated as (Stille et al. 1989):

$$\sigma_{\mathrm{re}} = \frac{2}{1+k}(p_0 + a) - a \tag{30}$$

and r_{e} as:

$$r_{\mathrm{e}} = r\left[\frac{\sigma_{\mathrm{re}} + a}{p_{\mathrm{i}} + a}\right]^{\frac{1}{k-1}}, \tag{31}$$

in which

$$k = \tan^2\left(45 + \frac{\varphi}{2}\right) \tag{32}$$

and

$$a = \frac{c}{\tan\varphi}. \tag{33}$$

c is the cohesion of the rock mass. As soon as plastic behavior has been induced, the radial deformations of the tunnel periphery are no longer u_{ie} but instead plastic radial deformations of the tunnel periphery, u_{ip}. u_{ip} can be calculated as:

$$u_{\mathrm{ip}} = \frac{rA}{f+1}\left[2\left[\frac{r_{\mathrm{e}}}{r}\right]^{f+1} + (f-1)\right], \tag{34}$$

where

$$A = \frac{1+\nu}{E}(p_0 - \sigma_{\mathrm{re}}) \tag{35}$$

and

$$f = \frac{\tan\left(45 + \frac{\varphi}{2}\right)}{\tan\left(45 + \frac{\varphi}{2} - \psi\right)}. \tag{36}$$

ψ is the dilatancy angle of the rock mass.

As excavation progresses passed the considered cross section, the distance x from the cross section to the excavation face increases. For small values of x, i.e. when the excavation face is close to the considered cross section, the undisturbed rock mass in front of the excavation will partly support the tunnel periphery. This is usually referred to as a fictitious supportive pressure that limits deformations. However, this fictitious supportive pressure decreases as the excavation progresses. Eventually, the fictitious supportive pressure does not counteract the deformation and thereby the maximum deformation, u_{final}, will be reached. The development of deformations follows a non-linear relationship (Fig. 4.5) as (Chang 1994)

$$u_{\mathrm{x}} = u_{\mathrm{final}}\left[1 - \left(1 - \frac{u_0}{u_{\mathrm{final}}}\right)\left(1 + 1.19\frac{x}{r_{\mathrm{e,max}}}\right)^{-2}\right], \tag{37}$$

in which $r_{\mathrm{e,max}}$ is the maximum radius of the plastic zone.

When the face of the excavation reaches the considered cross section, approximately one third of the final deformation expected for an unsupported tunnel has developed. The following relationship can be used to describe the magnitude of this deformation (Chang 1994):

$$u_0 = 0.279\left(\frac{r_{\mathrm{e}}}{r}\right)^{0.203u_{\mathrm{ie}}}. \tag{38}$$

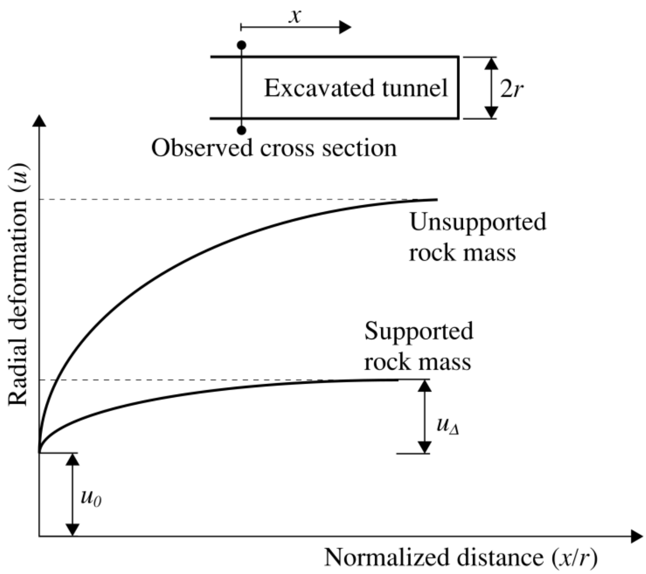

Figure 4.6: Development of deformation of the tunnel periphery during excavation for an unsupported and supported rock mass.

To limit deformations, different support measures can be utilized. Regularly, the support is illustrated by a separate support curve that crosses the ground–response curve at some particular deformation, i.e. the final supportive deformation. One available support measure for limiting of deformations is shotcrete. The response curve for a shotcrete support can be calculated as (Stille et al. 1989)

$$p_{\mathrm{i}} = k_{\mathrm{c}} \Delta u_{\mathrm{s}}, \tag{39}$$

where Δu_{s} is the deformation of the shotcrete and k_{c} is the stiffness of the shotcrete, given by

$$k_{\mathrm{c}} = \frac{E_{\mathrm{c}}}{r} \frac{r^2 - (r - t_{\mathrm{s}})^2}{(1 + \nu_{\mathrm{c}})[(1 - 2\nu_{\mathrm{c}})r^2 + (r - t_{\mathrm{s}})^2]}, \tag{40}$$

in which t_{s} is the shotcrete thickness. The relationship given in Eq. 43 is valid until the maximum pressure capacity of the shotcrete, $p_{\max}$ (Fig. 1) is reached. $p_{\max}$ can be calculated as

$$p_{\max} = \frac{1}{2}\sigma_{\text{cs}}\left[1 - \frac{(r - t_{\text{s}})^2}{r^2}\right], \tag{41}$$

where the σ_{cs} is the uniaxial compressive strength of the shotcrete.

Other support measures than shotcrete also exist. However, the effect of support measures such as rock bolts can be viewed upon differently depending on the type of bolt installed, i.e. incorporate the effect of the rock bolts into the ground response curve (Stille et al. 1989) or using a separate support curve, similar to the above shown (Hoek & Brown 1980). Therefore the reader is referred to one of the many books and peer-reviewed journal papers written on the subject (e.g. Brown et al. 1983, Carranza-Torres & Fairhurst 2000, Hoek & Brown 1980, Stille et al. 1989) for recommendations on how specific support measures can be incorporated into the convergence-confinement method.

5 Design of rock support with reliability-based methods

The utilization of reliability-based design methods in rock tunnel engineering has to some extent been addressed earlier. One of the early contributors to the subject was Kohno (1989). Kohno performed relatively extensive work over a large span of areas covering topics of both type I and type II, such as reliability of tunnel support in soft rock, reliability of tunnel lining in jointed hard rock, probabilistic evaluation of tunnel lining deformation through observation, and reliability of systems in tunnel engineering. Other contributors to the field have mainly contributed to the analysis of either type I or type II limit states. In this chapter a review of some of the performed work in both types of limit states is performed.

5.1 Limit states with separable load and resistance

For limit states of type I, the main work is found in the analysis of rock wedges, both in slopes and tunnels. As an example, Quek & Leung (1995) analyzed the reliability of a rock slope using the first-order second-moment method, complementing it with Monte Carlo simulations. On the same subject, Low (1997) analyzed sliding stability of a rock wedge in a rock slope. Low used an Excel spreadsheet and second-moment reliability indexes with both single and multiple failure modes to calculate the probability of sliding failure of a rock wedge. Similarly to the work performed by Low (1997), Jimenez-Rodriguez & Sitar (2007) analyzed the stability of a rock wedge using both FORM and Monte Carlo simulations in a system reliability analysis for a number of failure modes. The analysis showed that the results from Monte Carlo simulations could be approximated using FORM.

To study how clamping forces, the half-apical angle, and other parameters affect the calculated partial factors and results of a stability analysis, Bagheri (2011) used both deterministic and reliability-based methods. The results show that partial factors needed for a safe design are very sensitive to the half-apical angle and that they change significantly from case to case. Similar results are presented in Paper A. Similar to Bagheri (2011), Park et al. (2012) combined deterministic calculations and reliability-based methods to derive an equation for the *SF* of rock wedge failure in a slope and combined it with the point estimate method to calculate the probability of limit exceedance. Further, Low & Einstein (2013) compared the results from a reliability analysis of tunnel roof wedges and forces in rock bolts, using mainly FORM and second-order reliability method (SORM) against deterministic calculations and Monte Carlo simulations.

As can be seen from the previously performed work on reliability-based methods for limit states of type I they have been used successfully by a number of authors for a number of different limit states. However, only a limited amount of work have been found regarding the design of tunnel support, such as shotcrete or lining, subjected to loads from loose rock wedges. In addition, the magnitude and uncertainty of input variables are in many cases assumed.

5.2 Limit states with interaction between load and resistance

For limit states of type II, Laso et al. (1995) studied the probability of limit exceedance for tunnel support using the convergence–confinement method with four limit definitions: excessive support lining pressure, soil deformation, lining deformation, and critical strain of lining.

Celestino et al. (2006) used load and resistance factor design for two failure modes considered in design of shotcrete support. The failure modes used were: bearing capacity of the support footing for the shotcrete arch and failure of the shotcrete lining support. The analysis was performed based on a case study of a tunnel in Brazil with a railway crossing just above the tunnel.

Similar to the work that Bagheri (2011) and Park et al. (2012) performed for rock wedges, Nomikos & Sofianos (2010) developed an approach to use the *SF* in a probabilistic way. The developed method was utilized in two design situations: stability of rock pillars and stability of underground roofs in a layered rock mass.

As for rock wedges, Low has contributed quite significantly to the work performed in limit states of type II. First, Li & Low (2010) used FORM and the convergence–confinement method to perform a reliability-based analysis with two limit criteria: one for the rock mass and one for the shotcrete support lining. Secondly, Lü & Low (2011) performed a similar analysis but used SORM and the response surface method instead of FORM. Both results were compared to results from Monte Carlo simulations. Lastly, Lü et al. (2011) extended the previously performed analysis with a third limit criterion: a requirement that the length of the rock bolt must exceed the radius of the plastic zone minus the radius of the tunnel with at least 1.5 m. Similarly to Lü & Low (2011), the response surface method was used.

Zhang & Goh (2012) used empirical relationships and RMR ratings to estimate parameters for a numerical analysis of a rock cavern in the numerical modeling software FLAC3D. Using a $2k$-factorial design approach (k being the number of variables) in the analysis, frequency distributions for *SF* and strain were obtained. Based on the results, a regression model that could be used to calculate the probability of limit exceedance in a tunnel was developed. In the analysis, both ultimate and serviceability limit states were considered. Goh & Zhang (2012) also used artificial neural networks (ANN) combined with FLAC3D to study the *SF* in a tunnel.

Langford & Diederichs (2013) used numerical analysis combined with a modified point estimate method to analyze and discuss shotcrete support design. The analysis was performed in a case study of the Yacambú–Quibor tunnel.

Similar to the work performed for limit states of type I, reliability-based methods have been used successfully by a number of authors for a number of different limit states of type II; however, the previously performed work mainly concerns the behavior and design of the final support. The behavior of the tunnel during construction and the design of the preliminary or temporary support, or the final support during construction, have only been studied to a limited extent.

5.3 Reliability-based methods and the observational method in rock engineering

One of the main contributor to the work performed on the subject of reliability-based methods within the framework of the observational method applied in rock engineering is Spross, e.g. (Spross 2016, 2014, Spross et al. 2014b, Spross et al. 2016, Spross & Larsson 2014, Spross & Johansson 2017). Spross (2016) covers a wide spectrum of applications such as groundwater leakage control in tunnels, pore-pressure measurements in safety assessments of dams, and pillar stability in underground caverns. The main contribution from Spross (2016) is the presentation of a probabilistic framework for the observational method, which combines reliability-based design with Bayesian statistical decision theory. Other contributors to reliability-based design and the observational method include e.g. Stille et al. (2005b), Holmberg & Stille (2007), Holmberg & Stille (2009), Zetterlund et al. (2011).

6 Summary of appended papers

6.1 Paper A: Challenges in applying fixed partial factors to rock engineering design

William Bjureland, Johan Spross, Fredrik Johansson, Anders Prästings & Stefan Larsson

Accepted by *the Geo-Institute (Geo-Risk) 2017, Denver, Colorado, USA, 4-7 June 2017.*

This paper addresses some challenges of applying the fixed partial factors, suggested by Eurocode, to rock engineering design for a common limit state, i.e. the design of shotcrete against a loose block with adhesion in the shotcrete–rock interface. The paper illustrates, through a calculation example, how statistically calculated partial factors vary with a change in center to center distance between rock bolts. The varying partial factors highlight the predicament of using fixed partial factors in design of rock tunnel support since the relationship between the load and the resistance will vary with geometric changes. Thereby, using fixed partial factors, which assumes a constant relationship between the load and the resistance is inappropriate. The paper concludes that reliability-based methods would be advantageous in rock engineering design.

6.2 Paper B: Some aspects of reliability-based design for tunnels using observational method (EC7)

William Bjureland, Johan Spross, Fredrik Johansson & Håkan Stille

In: *Kluckner S, ed. EUROCK 2015. 1st ed. Salzburg, Austria; 2015:23-29.*

In this paper, an outline of a methodology for utilization of deformation measurements to predict the final radial deformation of the tunnel periphery and assess the probability of limit exceedance is presented. The proposed methodology fulfills the requirements of the observational method, as defined in Eurocode 7, partly by applying reliability-based methods within the framework of the observational method. The methodology is illustrated through a fictive calculation example in which Monte Carlo simulations are used to calculate the probability of limit exceedance in the preliminary design. The results from the Monte Carlo simulations are then updated during construction through a regression analysis and extrapolation along with Bayesian updating. The paper concludes that although only a simplified fictive calculation example was presented, the potential of the methodology is shown due to the fact that the requirements of the observational method, as stated in Eurocode 7, were possible to fulfill.

6.3 Paper C: Reliability aspects of rock tunnel design with the observational method

William Bjureland, Johan Spross, Fredrik Johansson, Anders Prästings & Stefan Larsson

Submitted to *International Journal of Rock Mechanics and Mining Sciences*.

In this paper, the deformation capacity of a shotcrete support is included in the assessment of the probability of limit exceedance. Similarly to Paper B, the paper focuses on showing how the requirements of the observational method, as defined in Eurocode 7, can be fulfilled through application of reliability-based methods. A similar calculation example as in Paper B is performed but an extension is made to include the capacity of the shotcrete support, correlation between input parameters, and a distributed maximum allowable deformation related to the deformation capacity of the shotcrete. The paper concludes that the combination of reliability-based methods and the observational method strengthens the structural safety considerations in rock tunnel engineering design. Also, the utilization of predictions of a future behavior along with a Bayesian updating procedure allows for an optimization of decision making during construction.

7 Discussion on the applicability of reliability-based design of tunnel reinforcement

7.1 The partial factor method

As illustrated in Paper A, using the partial factor method with fixed partial factors, as proposed by Eurocode, in design of support for underground excavations in rock presents some challenges. As can be seen in the paper, the magnitude of the calculated partial factors varies significantly with a change in the geometry of the problem. The reliability level of the structure changes depending on the geometry of the limit state analyzed and thereby different levels of safety are achieved for similar limit states with different geometric layouts. This implies that the fixed partial factors do not work as originally intended, as argued for in Paper A.

As also can be seen in Paper A, the original version of the partial factor method, described in Section 2.3.2, has the potential to stringently account for statistical and physical uncertainty of variables, sensitivity of the structural system to these variables, and also the target reliability index, β_T. Therefore, it can be applied as a safety assessment method in geotechnical design by calculation for limit states of type I, in which a distinction, after simplifications, can be made between the parameters affecting the load and the parameters affecting the resistance. However, when using the partial factor method it might be difficult to stringently account for the epistemic uncertainties, such as model and prediction uncertainty, present in rock engineering design. In addition, the engineer must be able to quantify the magnitude and uncertainty of input variables and describe them in terms of a distribution. Therefore, using the partial factor method as a safety assessment method in design of underground excavations in rock is inappropriate in design situations that do not mainly incorporate well quantified aleatory uncertainty, such as statistical and physical uncertainty of input variables. Further, to calculate partial factors for every design situation might require more work than simply applying reliability-based design methods, such as Monte Carlo simulations or FORM, directly.

For limit states of type II, such as the one presented in Paper B and Paper C, a clear distinction cannot be made between the parameters affecting the load and the parameters affecting the resistance. The partial factor method is therefore not an applicable safety assessment method for these types of limit states.

7.2 Reliability-based methods and the observational method

As shown in Paper B and Paper C, reliability-based methods have the ability to stringently account for uncertainties in input variables and the effect that specific parameters have on the analyzed limit state function. As can be seen in the papers and the presentation made in Chapter 4, depending on the type of the analyzed limit state, type I or type II, and on the questions the engineer is faced with, different reliability-based methods are available for usage in design of support for underground excavations in rock. For simple limit states of type I with linear limit state functions, simplified methods such as FORM might be suitable. For more complex non-linear limit states of type II with complex distributions of input variables, methods such as Monte Carlo simulations might be preferable. Regardless of the analyzed limit state, different reliability-based methods are available as safety assessment methods, when performing geotechnical design by calculation, to assure that the designed underground structure fulfills society's demand on acceptable levels of safety. However, one disadvantage of using reliability-based methods is that, similar to the original version of the partial factor method, it might be difficult to stringently account for epistemic uncertainty such as phenomenological, decision, model, prediction, and human factors.

Therefore, geotechnical design by calculation, using reliability-based methods for the safety assessment, should, as discussed in Paper B and Paper C, preferably, be performed within the framework of the observational method. Doing so, the reliability-based methods can act as a means to stringently account for physical and statistical uncertainties and as a basis for decision making. In addition, observations performed during construction can be utilized to account for and reduce uncertainties related to phenomena's, predictions, and human factors. Lastly, the analyzed structure can be continuously assessed, based on the results of the performed observations, using the reliability-based methods to verify that the structure fulfills society's demands on acceptable levels of safety. Thereby environmental and economical optimization of a structure can be pursued, without compromising on the required levels of safety. However, the cost and time of design, monitoring, and preparation of contingency actions must be considered when making the choice of using reliability-based methods within the observational method instead of using design by calculation (Spross & Johansson 2017). Also, for reliability-based calculations, within the framework of the observational method, some challenges still exist.

One of the most challenging parts, as discussed in all of the appended papers, is to define what “failure” actually means. Is failure the limit where limit exceedance causes a section of, or the whole, tunnel system to collapse? Or is it maybe the plastic limit of one of the components included in the analyzed system? As discussed in e.g. Johansson et al. ((In press)), the issue of defining failure is by no means only relevant when analyzing a tunnel using reliability-based methods. The same problem is present regardless of the method chosen for the analysis. Therefore, as suggested by e.g. Mašín (2015) it might be more appropriate to define the limit as a limit of “unsuccessful behavior”, the definition applied in this thesis, instead of failure. However, if that approach is applied in a common practical design situation a question arises of what β_T the engineer should use in order to fulfill society’s demanded levels of safety.

As mentioned in Paper A, another question arises when using reliability-based methods in combination with the limit states used in today’s design practice (e.g. Lindfors et al. 2015). Are the limit states and calculation models calibrated to be used with reliability-based methods? Consider for example the limit state presented in Paper A, classically the analysis has been performed with the assumption that the block exists between the rock bolts. However, if reliability-based methods are to be used along with β_T values presented in the Eurocodes (CEN 2002), the engineer needs to consider aspects such as the probability that the block exists, that it will be located between rock bolts, and that it is actually loose. This highlights the limited knowledge that we have in the correctness of our calculation models.

Lastly, tunnel engineering has historically included only a small amount of tests performed in a single project. However, when applying reliability-based methods the engineer must be able to define the relevant input variables in terms of distributions, which usually require that a large amount of tests are performed. Therefore, as suggested in Paper A, incorporating experience-based data when performing reliability-based calculations might be necessary.

8 Conclusions and suggestions for future work

When analyzing rock tunnel support and verifying that a structure fulfills society's demand on acceptable levels of safety, it is important that the method used accounts for uncertainties in a consistent and stringent manner. The aim of this study was to identify possibilities and practical difficulties of using the partial factor method and reliability-based methods, exclusively or in combination with the observational method, for design of rock tunnel support. The review performed in this thesis has highlighted some important aspects of these methods applied to design of underground excavations in rock.

As discussed in the thesis and the appended Paper A, the Eurocodes' version of the partial factor method can be questionable to use as a design method for rock tunnel support. The original version of the partial factor method can be used in limit states of type I, in which a relatively clear distinction between the load and the resistance can be made. However, the partial factor method mainly accounts for aleatory uncertainties while uncertainties present in rock engineering design are to a large extent epistemic. Therefore, using geotechnical design by calculation accompanied by a safety assessment with the partial factor method for design of underground excavations in rock is not suitable.

Reliability-based methods have, similar to the original version of the partial factor method, the ability to stringently account for uncertainties in parameters involved in design of support for underground excavations in rock. However, similar to the partial factor method, reliability-based methods are best suited to account for aleatory uncertainty. Therefore, design of underground structures in rock should preferably be performed using reliability-based methods, as a basis for decision making, within the framework of the observational method. Thereby, both epistemic uncertainties, through the use of the observational method, and aleatory uncertainties, through the use of reliability-based methods, can be accounted for in design of underground excavations in rock. However, as can be seen from the review performed in this thesis and the work performed in the appended papers further efforts are still needed within the field of reliability-based design methods for design of rock tunnel support. Some suggestions for future work are listed below:

- Further efforts need to be put into the definition of failure and how it relates to different limits of acceptable behavior.

Alongside this, it should be clarified what the defined β_T actually relate to.

- Information of parameters related to the design of underground excavations in rock, in terms of their representative distributions, is the basis for using reliability-based design. Therefore, further laboratory tests need to be performed along with gathering of data from constructed rock tunnels.

- A deeper review of the combination of using the limit states of today's practice in combination with reliability-based methods would be beneficial. Taking the limit state presented in Paper A as an example it was assumed in this review that the calculation model is appropriate for reliability-based design methods. However, the conditional probability that the block exists will play a crucial role in a reliability-based analysis of that particular limit state and therefore the calculation model might not be appropriate for the given β_T.

- Reliability-based methods have successfully been used within the framework of the observational method in the appended Paper B and Paper C. However, further studies on how to define acceptable limits of behavior and how these can be used in reliability-based design need to be further addressed.

- Reliability-based methods have been successfully used in combination with analytical calculations. In future studies reliability-based methods should be combined with numerical calculations.

- If the above stated future research questions are answered, it would be interesting to compare different safety assessment methods in a case study.

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